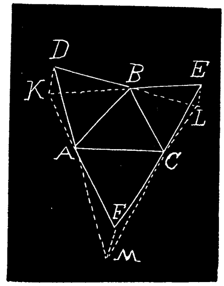
GEOMETRY.

106. Proposed by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, Ohio.

Upon the sides of any triangle ABC let the equilateral triangles ABD, BCE, and CAF be described, and let their exterior sides produced intersect, BE and AF in K, DB and FC in L, and DA and EC in M. Prove DK, EL, FM, parallel.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; J. SCHEFFER, A. M., Hagerstown, Md., and G. I. HOPKINS, Instructor in Mathematics and Physics, High School, Manchester, N. H.

Angle $KAB=180°-60°-$angle CAB, angle $ABK=180°-60°-$angle ABC.



$\therefore$ Adding, angle $KAB+$angle $ABK=240°-$angle $CAB-$angle ABC. Angle $AKB=180°-($angle $KAB+$angle $ABK)$.

$\therefore$ Angle $AKB=$angle $CAB+$angle $ABC-60°$.

Again, angle $BCL=180°-60°-$angle ACB. Angle $ACB=180°-($angle $CAB+$angle $ABC)$.

$\therefore$ Angle $BCL=$angle $CAB+$angle $ABC-60°$.

$\therefore$ Angle $BCL=$angle AKB.

Angle $MAC=60°+$angle MAF, and angle $KAB=60°+$angle KAD. $\therefore$ Angle $MAC=$angle KAB.

Similarly, angle $MCA=$angle BCL. $\therefore$ Angle $MCA=$angleAKB.

$\therefore$ Triangle AMC is similar to triangle AKB.

$\therefore$ $AC:AK::AM:AB$, or $AF:AK::AM:AD$.

$\therefore$ Triangle AKF is similar to triangle AKD.

$\therefore$ Angle $AMF=$angle ADK.

$\therefore$ KD is parallel to MF. Similarly EL is parallel to MF.

II. Solution by the PROPOSER.

Points K, F, C, B are concyclic. $\because$ $\angle CBE=\angle AFC$.

$\therefore$ $\angle BKA=\angle BCL$. $\angle LBC=\angle KBA$.

$\therefore$ Triangles AKB and CBL are similar, and $KB/AB=LB/CB$, or $KB/EB=LB/EB$.

$\therefore$ KD is parallel to EL.

Similarly $AM/AD=AD/AK$, and therefore FD is parallel to KD.

And therefore EL is parallel to KD is parallel to FM.

III. Solution by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.

Let ABC be any triangle having equilateral triangles described upon its sides, and their exterior sides produced to intersect BE and AF in K, DB and FC in L, and DA and EC in M. Join FM, DK and EL.

The triangles BCL and ACM are similar, hence $BC:CL::CM:CA$, or $CE:CL::CM:CF$. And since the $\angle ECL=\angle FCM$, the triangles CLE and CFM are similar and equiangular, the angle FMC being equal to the angle LEC.

$\therefore$ ER is parallel to FM..................................(1).

The triangles ABK and AMC are similar, hence $AB : AK :: AM : AC$, or $AD : AK :: AM : AF$.

Since the $\angle DAK = \angle MAF$, the triangles DKA and MFA are similar, and $\angle ADK$ is equal to $\angle AMF$.

$\therefore$ DH is parallel to FM.......................................(2).

$\therefore$ DK, FM and EL are parallel. Q. E. D.

IV. **Solution by CHARLES C. CROSS, Libertytown, Md.**

Draw the figure as indicated in the problem.

Let $\angle BLE = x$, $\angle CEL = y$, $\angle DKB = z$, $\angle ADK = w$, $\angle CEM = v$, and $\angle FMC = w$.

$\angle ECL = 180° - (120° + C) = 60° - C$.

Similarly, $\angle EBL = A - 60°$, and $\angle KAD = 60° - A$.

$\angle BCL = 180° - (60° + C) = 120° - C$.

Similarly, $\angle LBC = 120° - B$, and $\angle BAK = 120° - A$.

Hence $\angle BLC = B + C - 60°$, and $\angle BKA = B + A - 60°$.

$\angle BLE + \angle BLC + \angle CEL + \angle ECL = 180°$; by substitution $B + x + y = 180°$...(1).

$\angle BKA + \angle BKD + \angle KDA + \angle KAD = 180°$; by substitution $B + w + z = 180°$..(2).

From (1) and (2), $x + y = w + z$.......................................(3).

If EL and DK are parallel, angle DKB=angle BEL, and angle BLE= angle KDB, or $z = 60° + y$ and $x = 60° + w$. Substituting in (3), $60° + w + y = 60° + w + y$. Hence EL and DK are parallel.

Angle CFM+angle CMF+angle $FCL = 180°$; by substitut'n $v + w - C = 120°$..(4).

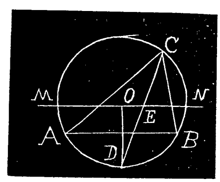
If EL and FM are parallel, then angle MFC=angle ELC, and angle EMC =angle CEL, or $v = x + A + C - 60°$, and $w = y$. Substituting in (4), $A + x + y = 180°$ Since by (1) this relation is true, hence EL and FM are parallel.

107. **Proposed by T. W. PALMER, A. M., Professor of Mathematics, University of Alabama.**

Construct a triangle, given base, vertical angle and radius of inscribed circle.

I. **Solution by H. C. WHITAKER, Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.**

Denote the base by AB, the vertex by C, and the incenter by O. The angle AOB equals $90° + \frac{1}{2}C$ and hence one locus for O is the arc of a segment capable of containing this angle. Another locus is a parallel to the base the in-radius away. Hence the incircle can be constructed ; AC and BC are then drawn tangent to it.



II. **Solution by J. SCHEFFER, A. M., Hagersfown, Md.**

Describe on the given base AB a circle the upper segment of which contains the given vertical angle. From the center O of this circle let fall the perpendicular on AB and produce it to D. At a distance from AB equal to the given radius of the inscribed circle draw MN parallel to AB. From D as a center with a radius equal to BD draw an arc cutting MN at E, connect E with D and extend DE until it